A Level Further Maths Year 2 Scheme of Work

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| **Unit** | **Week** | **Content** | **Resources** |
| Further Matrices | Week 1 | Inverse and determinant of a 3x3 MatrixPlane geometry | [**A2 Further Maths 1 - Inverse and determinant of a 3x3 Matrix**](https://youtu.be/omxBnqZz-Ik)[**A2 Further Maths 2 - Plane geometry**](https://youtu.be/x2n9dlK-FbM)**A2 Further Maths 2 - Plane geometry Matching** |
| Week 2 | Find eigenvalues and eigenvectors of 2×2 and 3×3 matricesFind and use the characteristic equation.Understand the geometrical significance of eigenvalues and eigenvectors | [**A2 Further Maths 3 - Eigenvectors and Eigenvalues**](https://youtu.be/U3Z_JqlM7no) |
| Week 3 | Diagonalisation of matricesWrite a matrix in the form $UDU^{-1}$ and understand $M^{n}=UD^{n}U^{-1}$ | [**A2 Further Maths 4 - Diagonalisation**](https://youtu.be/FovXghXvLzQ) |
| Further Vectors | Week 4 | Calculate and understand the properties of the vector product.Understand and use the equation of a straight line in the form $\left(r-a\right)×b=0$Use vector products to find area of a triangleUnderstand and use the vector and Cartesian forms of the equation of a planeExpress the equation of a plane in the form **r**.**n**=dCalculate the angle between two planes and the angle between a line and a planeFind the line of intersection of two planes or three planesFind the intersection of a line and a plane.Calculate the perpendicular distance from a point to a plane | [**A2 Further Maths 5 - Vector Product**](https://youtu.be/Pn4qqQEBw38)[**A2 Further Maths 6 - Equations of planes**](https://youtu.be/HOeaHeXiais) |
| Week 5 | [**A2 Further Maths 7 - Further Vectors Planes and Lines**](https://youtu.be/c999lnNkxjs) |
| Chi squared tests | Week 6 | Knowledge of when and how to apply Yates’ correction. | [**A2 Further Maths 8 - Yates' Correction**](https://youtu.be/UwkJ385nSrM) |
| DRVs and expectation | Know the formula $E\left(g\left(X\right)\right)=\sum\_{}^{}g\left(x\_{i}\right)p\_{i}$Find the mean, variance and standard deviation for functions of a DRV such as $E\left(5X^{3}\right)$, $E\left(18X^{-3}\right)$, $Var\left(6X^{-1}\right)$ | [**A2 Further Maths 9 - Discrete random variables**](https://youtu.be/AVPYyH9wfkY) |
| Type I and Type II errors | Week 7 | Teach A2 Normal Distribution including Hypothesis TestingCalculate probability of making Type I error from tests based on a normal distributionUnderstand the power of a test. Calculations of P(Type II error) and power for a test for tests based on a normal, Binomial or a Poisson distribution | [**A2 Further Maths 10 - Type I and Type II Errors**](https://youtu.be/UgWGHYM6wPU) |
| CRVs | Week 8 | Understand and use a cumulative distribution function, F(*x*). Know the relationship between f(*x*) and F(*x*).Understand the rectangular distribution.Know the conditions for the rectangular distribution to be used as a model.Calculate probabilities from a rectangular distribution.Know proofs of mean, variance and standard deviation for a rectangular distribution | [**A2 Further Maths 11 - Cumulative distribution function**](https://youtu.be/vHKYfnAxe40)[**A2 Further Maths 12 - Rectangular distribution**](https://youtu.be/4WPgOnRjCu4) |
| Half Term |
| Further Complex Numbers | Week 9 | Understand de Moivre’s theorem and use it to find multiple angle formulae and sums of seriesUse DMT for both finite and infinite trigonometric series.Know and use the definition $e^{iθ}=cosθ+isinθ$ and the form $z=re^{iθ}$ | [**A2 Further Maths 13 - De Moivre’s Theorem**](https://youtu.be/bmQFK7W2TCI)[**A2 Further Maths 14 - Complex numbers exponential form**](https://youtu.be/aAGAF1_H_sg) |
| Week 10 | Find the $n$ distinct $n$th roots of $re^{iθ}$ for $r\ne 0$ and know that they form the vertices of a regular $n$-gon in the Argand diagram.Use complex roots of unity to solve geometric problems | [**A2 Further Maths 15 - Roots of unity**](https://youtu.be/1jp9B8swA-o) |
| Graphs and conics | Week 11 |  Modulus of functions and associated inequalitiesGraphs of $y=|f\left(x\right)|$ and $y=\frac{1}{f(x)}$ for given $y=f(x)$Understand the concept of an oblique asymptoteConic functions extended to composite transformations including rotations (by multiples of 90 degrees). | [**A2 Further Maths 16 - Modulus function**](https://youtu.be/30Pgo2oe4wo)[**A2 Further Maths 17 - Reciprocal of a function**](https://youtu.be/uisePq7YLac)[**A2 Further Maths 18 - Graphing rational function oblique asymptotes**](https://youtu.be/-G5KncmMgyg)[**A2 Further Maths 19 - Transformations of conic functions**](https://youtu.be/OdhfmkANIik) |
| Hyperbolic functions | Week 12 | Use the domains and ranges of the hyperbolic functions.Understand the definitions of hyperbolic functions $sechx$, $cosechx$ and $cothx$, including their domains and ranges.Use the domains and ranges of the inverse hyperbolic functions.Differentiate and integrate hyperbolic functions.Be able to derive logarithmic formulae for sech, cosech and coth | [**A2 Further Maths 20 - Inverse and reciprocal hyperbolic functions**](https://youtu.be/svoIHEimG-s) |
| Week 13 | Understand and use the identities;* $sech^{2}θ=1-tanh^{2}θ$
* $cosech^{2}θ=coth^{2}θ-1$
* $cosh2θ=cosh^{2}θ+sinh^{2}θ$

$$sinh2θ=2sinhθcoshθ$$Construct proofs involving hyperbolic functions and identities | [**A2 Further Maths 21 - Hyperbolic functions calculus and identities**](https://youtu.be/uuwB6BWjKJQ) |
| Numerical methods | Week 14 | Mid-ordinate rule and Simpson’s rule for integration.Euler’s step by step method for solving first order differential equationsImprovement: $y\_{r+1}=y\_{r-1}+2hf(x\_{r},y\_{r})$, $x\_{r+1}=x\_{r}+h$Other improvements may be used but will be clear in exam question | [**A2 Further Maths 22 - Mid-ordinate and Simpsons rule**](https://youtu.be/zvmM4Bl9TwI)[**A2 Further Maths 23 - Euler’s step by step method**](https://youtu.be/aJwxwT4xBuc) |
| Exam | Week 15 | Winter Exam |  |
| Christmas |
| Further Calculus | Week 16 | Feedback on Winter ExamEvaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinityDifferentiate inverse trigonometric and hyperbolic functions | [**A2 Further Maths 24 - Improper integrals**](https://youtu.be/Q8Ws3LmB1zA)[**A2 Further Maths 25 - Differentiate inverse trigonometric functions**](https://youtu.be/_ePNtZOm_2Y) |
| Week 17 | Integrate functions of the form $(a^{2}-x^{2})^{-\frac{1}{2}}$ , $(a^{2}+x^{2})^{-1}$ , $(x^{2}+a^{2})^{-\frac{1}{2}}$ and $(x^{2}-a^{2})^{-\frac{1}{2}}$ and be able to choose trigonometric and hyperbolic substitutions to integrate associated functions.Integrate using partial fractions (extend to quadratic factors $ax^{2}+c$ in the denominator).Derivation and use of reduction formulae for integration.Find the Maclaurin series of a function including the general term. | [**A2 Further Maths 26 - Further integration**](https://youtu.be/4rB3OHn3E5E)[**A2 Further Maths 27 - Reduction formulae**](https://youtu.be/g_sYeb44VSk) |
| Week 18 | Evaluation of limits using Maclaurin series or l'Hôpital's rule.The limits $\lim\_{x\to \infty }(x^{k}e^{-x})$ and $\lim\_{x\to 0}(x^{k}lnx) $where $k>0$, applied to improper integralsArc length and area of surface of revolution for curves expressed in Cartesian or parametric coordinates.Find the area enclosed by a polar curve | [**A2 Further Maths 28 - Maclaurin series, limits and l'Hôpital's rule**](https://youtu.be/XERnGxF2Co4)[**A2 Further Maths 29 - Arc length and area of surface of revolution**](https://youtu.be/Mv8gKZG8t9k)[**A2 Further Maths 30 - Area of polar curves**](https://youtu.be/clHzEjR5rP4) |
| Differential Equations | Week 19 | Find and use an integrating factor to solve differential equations of form $\frac{dy}{dx}+P\left(x\right)y=Q(x)$ and recognise when it is appropriate to do so.Find both general and particular solutions to differential equations.Solve differential equations of form $y^{''}+ay^{'}+by=0 $ where $a$ and $b$ are constants by using the auxiliary equation.Solve differential equations of form$y^{''}+ay^{'}+by=f(x)$ where $a$ and $b$ are constants by solving the homogeneous case and adding a particular integral to the complementary function (in cases where $f(x)$ is a polynomial, exponential or trigonometric function).Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of solution of the differential equation | [**A2 Further Maths 31 - Differential equations integrating factor**](https://youtu.be/HM0gJ70V8XE)[**A2 Further Maths 32 - Differential equations 1st order**](https://youtu.be/hPEgNQjYL0k)[**A2 Further Maths 33 - Differential equations 2nd order**](https://youtu.be/qnKGNeemc6o) |
| Week 20 | Use differential equations in modelling in kinematics and in other contexts.Solve the equation for simple harmonic motion$\ddot{x}=-ω^{2}x$ and relate the solution to the motion.Note and learn formulae associated with SHM. Use of Hooke’s Law with $T=kx$ to formulate a differential equation for simple harmonic motion, where $k$ is a constant. | [**A2 Further Maths 34 - Modelling with differential equations**](https://youtu.be/PXnS3b0qqTU)[**A2 Further Maths 35 - Simple harmonic motion and Hooke's law**](https://youtu.be/Yq1_Zzv1QEI) |
| Week 21 | Model damped oscillations using 2nd order differential equations and interpret their solutions.Use models for damped motion where damping force is proportional to the velocity.Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled 1st order simultaneous equations and be able to solve them, for example predator-prey models | [**A2 Further Maths 36 - Damped harmonic motion**](https://youtu.be/FRmO3v9HyKE)[**A2 Further Maths 37 - Coupled differential equations**](https://youtu.be/pbILa5RJdk4) |
| Half Term |
| Exponential distribution | Week 22 | Know conditions for an exponential distribution to be used as a model.Know the probability density function, $f(x)$, and the cumulative distribution function, $F(x)$, for an exponential distributionCalculate probabilities for an exponential distribution using $F(x)$ or integration of $f(x)$Know proofs of mean, variance and standard deviation for an exponential distribution.Understand that the lengths of intervals between Poisson events have an exponential distribution | [**A2 Further Maths 38 - Exponential distribution**](https://youtu.be/51ExzMdO78k) |
| t distribution | Week 23 | Test for the mean of a normal distribution with unknown variance using a *t*-statistic with appropriate degrees of freedomConstruct confidence intervals, symmetric or otherwise, from small samples, for the mean of a normal distribution with unknown variance using the *t*-distribution. | **A2 Further Maths 39 - t-distribution** |
| Graph Theory | Week 24 | Kuratowski’s Theorem and isomorphism | **A2 Further Maths 40 - Kuratowski's theorem****A2 Further Maths 41 - Isomorphism between graphs** |
| Binary Operations and Group Theory | Week 25 | Understand and use the group axioms: closure, identity, inverses and associativity, including use of Cayley tablesUnderstand and use the language of groups including: ‘order’, ‘period’, ‘subgroup’, ‘proper’, ‘trivial’, ‘non-trivial’.Understand and use Lagrange’s theorem | **A2 Further Maths 42 - Language of groups** |
| Week 26 | Recognise and use finite and infinite groups and their subgroups, including: groups of symmetries of regular polygons, cyclic groups and abelian groupsIdentify and use the generators of groupsRecognise and find isomorphism between groups of finite order | **A2 Further Maths 43 - Types of groups, generators and isomorphism** |
| Easter |
| Network Flows | Week 27 | Find the value of a cut for a network containing arcs that have both upper and lower capacities.Use flow augmentation from an initial flow to find a maximum flow through a network with only upper capacitiesState the maximum flow through a network in the context of the problemFind a flow pattern corresponding to the maximum flow through a network.Be able to use flow augmentation from an initial flow to find a maximum flow through a network with lower and upper capacities | [**A2 Further Maths 44 - Network Flow Augmentation Algorithm and Student Sheet**](https://youtu.be/Kiqei9uuFjc)**A2 Further Maths 45 - Network Flows Maximum and Minimum Arcs and Student Sheet** |
| Simplex Algorithm | Week 28 | Use the Simplex algorithm for optimising (maximising and minimising) an objective function including the use of slack variablesInterpret a Simplex tableauConvert higher order games to linear programming problems and solve using the Simplex algorithm | **A2 Further Maths 46 - Simplex Algorithm****A2 Further Maths 47 - Game Theory and Simplex** |
| Gantt Charts | Week 29 | Construct and interpret Gantt (cascade) charts and resource histogramsCarry out resource levelling (using heuristic procedures) and evaluate problems where resources are restricted | **A2 Further Maths 48 - Gantt Charts and Resource Histograms** |